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# MODELLING OF AN INFLUENCE OF LIQUID VELOCITY ABOVE THE NEEDLE ON THE BUBBLE DEPARTURES PROCESS

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**Abstract:** In the present paper, the influence of liquid flow above the needle on a periodic or chaotic nature of the bubble departures process was numerically investigated. During the numerical simulations bubbles departing from the needle was considered. The perturbations of liquid flow were simulated based on the results of experimental investigations described in the paper [1]. The numerical model contains a bubble growth process and a liquid penetration into a needle process. In order to identify the influence of liquid flow above the needle on a periodic or chaotic nature of bubble departures process, the methods data analysis: wavelet decomposition and FFT were used. It can be inferred that the bubble departure process can be regulated by altering the hydrodynamic conditions above the needle, as variations in the liquid velocity in this area affect the gas supply system's conditions. Moreover, the results of numerical investigations were compared with the results of experimental investigation which are described in the paper [2]. It can be considered that, described in this paper, the numerical model can be used to study the interaction between the bubbles and the needle system for supplying gas during the bubble departures from two needles, because the interaction between the bubbles is related to disturbances in the liquid flow above the needle.

**Keywords:** bubble, liquid penetration into the needle, bubbles interaction

## 1. INTRODUCTION

The gas bubble flow and gas bubble formation process in liquids are found in: oceans, chemical processes, pharmaceutical industries or industrial equipment [3]. In medicine, ultrafine gas bubbles are used to transport medications [4]. The studies on bubble departure and interactions between bubbles serve as a preliminary exploration into the greenhouse effect caused by methane bubbles escaping in the oceans [5,6]. The gas bubble flow is investigated during the exploitation of the hydrocarbon deposits, too [7]. Moreover, the knowledge of bubble flow and bubble departure process is very important during the aeration or saturation process which helps e.g. in purifying the surface water or municipal sewage [8]. Regulating the bubble departure process can enhance mass transfer in the bubble column. [9,10,11]. The investigations of interaction between bubbles is treated as an introduction to investigations of the bubble formation process during boiling [12,13,14].

There are papers, in which researchers describe their results of investigations of single, double or multi bubble departures. In these investigations bubbles are generated from single [15-20] and twin or more needles or orifices [23-30].

The bubble departure time can be split into bubble growth time and bubble waiting time [15-19]. At relatively low gas flow rates, the liquid penetrates the needle or orifice during the bubble waiting time. The liquid penetration into the needle or orifice is connected with a decrease in gas pressure occurrences in the system for supplying gas [15-19] and it is modified by liquid pressure changes above the needle caused by perturbations in liquid flow in the needle neighbourhood [2]. In other papers [19-21], the chaotic nature of bubble departures process, phenomena which influence chaotic bubble departures or bubbles trajectories were investigated. In the paper [20] it was demonstrated that the chaotic nature of bubble trajectories is due to the shape of the departing bubbles and the liquid flow induced by the moving bubbles within the bubble column. In papers we can find that two groups of phenomena are responsible for the chaotic nature of bubble behaviours: the first group is connected with the bubble interface oscillations, liquid flow around the needle or orifice and the second group is connected with the processes which appear in the gas supply system of the needle [16,21,22].

The process of bubble departures from due or more needles or orifices was investigated in papers [23-31]. In papers [23,24] it was shown that interactions between bubbles, departed from twin needles can lead to synchronous or alternative bubble departures, bubble coalescence or bubble bouncing. The bubble coalescence or bubble bouncing depend on the bubble Reynolds numbers. The impact of the distance between needles on bubble interactions in selected kinds of liquids (with different physical and chemical properties) was investigated in paper [25]. In the paper [2] the interaction between bubbles or bubbles and the system for supplying gas was investigated based on the process of liquid penetration into the needle. It has been demonstrated that hydrodynamic interactions can result in periodic, chaotic or multi-period changes of liquid movement inside the needle gas supply system. Those changes influence the nature of the bubble departures process [2]. In the paper [25] the regimes of alternative bubble departures (ABD coefficient) in different kinds of liquids are proposed. The coefficient ABD comprises the distance between needles, air volume flow rate, liquid properties, and the frequency of bubble departures. The regimes of synchronous bubble departures from twin orifices are investigated in the paper [26]. Fur-

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thermore, the ABD coefficient was examined for orifices in this study [26]. Additionally, research on bubble interactions was conducted in papers [26-29]. In another study [20], bubbles were generated in water and an aqueous glycerine solution, revealing that such interactions alter bubble trajectories. The study concluded that bubble interactions and coalescence depend on gas flow rates, tube spacing, and liquid properties. The interaction of a bubble pair in viscoelastic shear-thinning fluids was experimentally investigated in paper [33]. It was suggested the elasticity and deformability are responsible for interaction between the bubble pair like in non-Newtonian fluids.

The numerical models of bubble departures (for example, proposed in the papers [31,32]) are very sensitive to changes in boundary conditions and modifications to systems of differential equations. Consequently, modelling the interaction between bubbles departed from twin needles or the influence of bubble departure from one needle on the process of bubbles departing from neighbouring needles is very difficult or impossible. Based on results of experimental investigation of liquid changes caused by departed bubbles (presented in the paper [1]), in present paper the modelling of the perturbations of bubble departures process caused by liquid flow above the needle is proposed. Proposed model can be used to investigate bubble and gas supply system interaction. The results of numerical investigation are consistent with the results of experimental investigation described in paper [2], where the perturbations of liquid flow above the needle are caused by growing and moving bubbles.

In the present paper the influence of liquid flow above the needle (according to the modifications caused by departed and moving bubbles) on the bubble departure process was numerically investigated. For modelling the interaction the model of the bubble growth and liquid movement inside the needle, which has been proposed in paper [32] was used. The model was modified so that it was possible to change the liquid inflow frequency and speed of liquid flow above the needle from which the bubbles are generated. In order to identify the influence of liquid flow above the needle on the bubble departure process, the wavelet decomposition and FFT methods were used. To investigate how liquid flow interactions affect disturbances in the gas supply system, manifesting as fluctuations in the depth of liquid penetration into the needle, 3D attractors were reconstructed. The study demonstrated that regulating the bubble departure process can be achieved by adjusting the hydrodynamic conditions above the needle. This is because variations in liquid velocity above the needle directly impact the conditions within the gas supply system. Described in this paper, the numerical model can be used to study the interaction between the bubbles and the needle gas supply systems during the bubble departures from two needles, because the interaction between the bubbles is related to disturbances in the liquid flow above the needle.

The structure of the paper is as follows. The numerical model is described in Chapter 2. Results of the numerical data analysis are shown in Chapter 3 -"Results of numerical investigations". A summary of the obtained results is shown in the "Conclusion" section.

## 2. DESCRIPTION OF NUMERICAL MODEL

For modelling the influence of liquid flow perturbations above the needle on the nature of the bubble departure process, the model of the bubble growth and liquid movement inside the needle was used. The model was proposed in papers [29,30], but it was modified so that it was possible to change the liquid inflow frequency and speed of liquid flow above the needle from which the bubbles are generated. These liquid flow perturbations correspond to the hydrodynamic interaction between the bubbles and the needle's gas supply systems. In the considered numerical model, attempts were made to maintain the same hydrodynamic conditions as in the experimental studies presented in paper [2]. The model was prepared in the SciLab environment, in which the equations described below were solved using the function ODE. The schema of bubble growth and liquid movement into the needle model is shown in Fig.1.

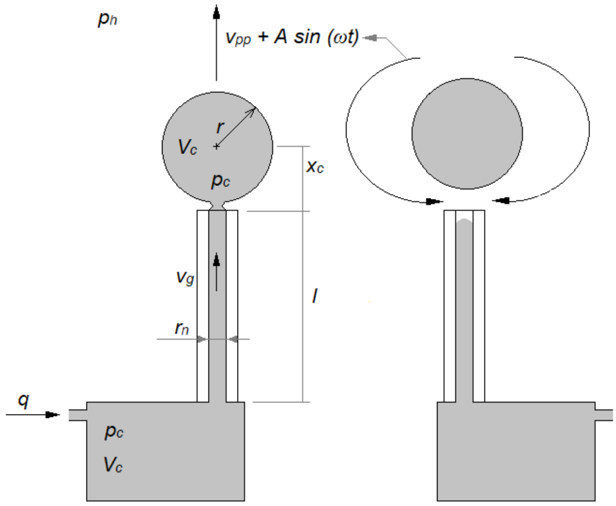


**Fig. 1.** The schema of bubble growth and liquid movement into the needle model

During the bubble growing stage an isothermal process was considered while the bubble growth was described by the Rayleigh–Plesset equation [30]:

$$r_b\left(\frac{d^2r_b}{dt^2}\right)-\frac{3}{2}\left(\frac{dr_b}{dt}\right)^2=\frac{1}{\rho_l}\left(p_b-p_h-\frac{2\sigma}{r_n}-\frac{4\mu_l dr_b}{dt}\right) \quad (1)$$

where: $r_b$ is the radius of the bubble (m), $\rho_l$ is the liquid density (kg/m$^3$), $p_b$ is the air pressure in bubble (Pa), $p_h$ is the hydrostatic pressure (Pa), $\sigma$ - the surface tension (N/m), $r_n$ is the inner radius of the needle (m), $\mu_l$ is the dynamic viscosity of the liquid (kg/ms).

The air volume flow rate supplied to the bubble through the needle was determined by the Hagen–Poiseuille equation:

$$\frac{dp_b}{dt}=\left(\frac{p_b}{V_b}\right)\left[\left(\frac{\pi}{8\mu_g}\right)\left(\frac{r_n^4}{l}\right)(p_c-p_b)-\frac{dV_b}{dt}\right] \quad (2)$$

where: $V_b$ is the volume of bubble (m$^3$), $\mu_g$ is the dynamic viscosity of the gas (kg/ms), $l$ is the needle length (m), $p_c$ is the air pressure in the gas supply system (Pa).

Pressure changes in the air supply system are described by the following equation:

$$\frac{dp_c}{dt}=\frac{k_c p_c}{V_c}\left[q-\frac{\pi r_n^4}{8\mu_g l}(p_c-p_b)\right] \quad (3)$$

where: $V_c$ is the gas supply system volume (m$^3$), $q$ is the gas volume flow rate supply to the needle (m$^3$/s).

Moreover, the model of the bubble growth stage accounts the forces acting on a growing bubble [30]:

– drag force:

$$F_d = 0.5C_d \rho_l \pi r^2 \left(\frac{dx_c}{dt} - v_{pp}\right)\left|\frac{dx_c}{dt} - v_{pp}\right| \quad (4)$$

– buoyancy force:

$$F_B = g(\rho_l - \rho_g) \cdot V_b \quad (5)$$

– maximum value of the surface tension force:

$$F_\sigma = 2\pi\ r_n \sigma \quad (6)$$

– added mass force:

$$F_{AM} = -\rho_l \frac{d}{dt}\left[C_M V_b \left(\frac{dx_c}{dt} - v_{pp}\right)\right] \quad (7)$$

– gas momentum:

$$F_M = \rho_g \frac{q_b^2}{\pi r_o^2} where\ \ q_b = \left(\frac{\pi}{8\mu_g}\right)\left(\frac{r_n^4}{l}\right)(p - p_b) \quad (8)$$

where: $g$ is the gravitational acceleration (m/s$^2$), $r_o$ is the bubble diameter (m), $q_b$ is the air flow rate supplied to the bubble (m$^3$/s), $Cd$ is the drag force coefficient, $CM$ = 0.5 is the added-mass coefficient for a sphere, $v_{pp}$ is the velocity of the liquid around the growing bubble (m/s) and $x_c$ is the position of the bubble centre (m).

The liquid movement inside the needle is described by the equation of motion of the liquid mass centre [32]:

$$\frac{d}{dt}\left\{\left[0.5\rho_l \pi r_n^2 x_l + \rho_l \frac{4}{3}\pi(2r_n)^3\right]\frac{dx_l}{dt}\right\} = F_1 - F_2 \quad (9)$$

The force $F1$ is related to the pressure difference that occurs in the system:

$$F_1 = -s\Delta p = -\pi r_n^2\left[p_g - \left(p_h + \rho_l g(2x_l) + A \cdot 2\frac{\sigma}{r_n} - \frac{\rho v_{pp}|v_{pp}|}{2}\right)\right] \quad (10)$$

The force $F2$ is related to the resistance of the movement of the liquid in the needle:

$$F_2 = 2 * 8\pi\mu_l x_l \frac{dx_l}{dt} \quad (11)$$

The corresponding pressure changes in the air supply system are described by the following equation [32]:

$$\frac{dp_c}{dt} = \frac{p_c}{V_c}\left(q + \pi r_n^2 \frac{dx_l}{dt}\right) \quad (12)$$

where: $x_l$ is the height of the liquid penetration into the needle (m) and $s$ is the cross-sectional area of the needle (m$^2$).

Criterion for the end of the liquid movement is correlated with the depth of liquid penetration – $x_l < 0$.

Based on the above equations, it can be assumed that, the perturbations in liquid flow velocity above the needle affect the bubble waiting time ($v_{pp}$ – Eq.10) or bubble growing time (Eq.4 and Eq.7) and consequently the perturbations in liquid flow velocity can modify the nature of periodic or chaotic bubble departure process. The character of changes of liquid flow above the needle is the same as liquid pressure changes above the needle. Therefore, based on the results of experimental investigations of liquid pressure changes modification above the needle cause by bubble departure process (being a periodic function), presented in paper [1] (Fig.2), the variations in liquid flow were approximated using a sinusoidal function, and based on equations (Eq.9, Eq.11, Eq.14), the velocity of the liquid above the needle was computed as:

$$v_{pp} = v'_{pp} + Asin(\omega t) \quad (13)$$

$V_{pp}'$ is the liquid flow above the needle induced by bubbles departing from it, focusing on the study of liquid movement in this process and $Asin(\omega t)$ is disturbances in the liquid velocity.

The changes of pressure above the needle, during the bubble departures (Fig.2).

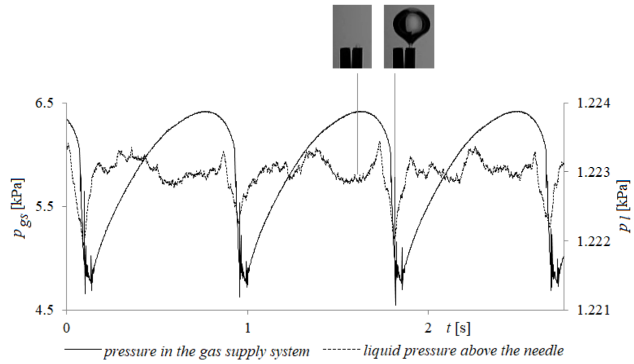


**Fig. 2** The changes of liquid pressure above the needle, during the bubble departures [1]

In the paper [1] it was shown that subsequent stages of bubble growth change the hydrodynamic conditions around the needle. The measured fluctuations in the liquid pressure above the needle, caused by bubble departures, are periodic. The greatest pressure changes are observed during the bubble growing time (Fig.2). During the bubble waiting time only slight fluctuations in liquid pressure above the needle are observed.

## 3. RESULTS OF NUMERICAL INVESTIGATIONS

In numerical simulations, the time series of liquid penetration into the needle during successive cycles of bubble departure, and the time series of liquid flow velocity above the needle, were analyzed. The simulations aimed to capture periodic and chaotic behaviors in the time series, depending on the frequency of variations in liquid flow velocity above the needle. Fig. 3 illustrates the time series of liquid penetration into the needle and disturbances in liquid velocity.

The model exhibits high sensitivity to boundary conditions; therefore the initial minimum value of pressure in the system for supplying gas was set at 5.4 kPa. The air volume flow rate was set as 0.006 l/min. The amplitude (A) was set as 0.1 and it simulates the velocity of liquid flow perturbations. Frequency of liquid perturbations was modified by changes in ωt and frequencies were changed during the simulations. For presented considerations the frequency was equal to 10.02 Hz, 11.52Hz and 11.9 Hz.

In order to investigate the repeatability of time series of liquid penetration into the needle, the 3D attractors were used. The 3D attractor is reconstructed using the stroboscope coordination. In this method, The coordinates of attractor points are determined by computing the positions based on sampled data points where the distance between them equals the time delay (τIf the subsequent trajectories on the attractor are closely spaced, the signal is considered quasi-periodic. However, if the trajectories in the attractor reconstruction begin to diverge from each other, it indicates that the analyzed signal exhibits chaotic behavior. The reconstruction of the 3D attractor, the time delay ($\tau$) was calculated for all time series separately. To determine τ, the mutual information method was used [32-34]. The first minimum of the function is treated as

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the proper value of $\tau$:

$$I(x_i, x_{i+\tau}) = \sum_{x_{i+\tau}} \sum_{x_i} p[x_i, x_{i+\tau}] \log_2 \left\{ \frac{p[x_i, x_{i+\tau}]}{p[x_i], p[x_{i+\tau}]} \right\} \quad (14)$$

where: $p[x_i, x_{i+\tau}]$ is the joint probability function of $\{x_i\}$ and $\{x_{i+\tau}\}$ , $p[x_i]$ and which are the marginal probability distribution functions of $\{x_i\}$ and $\{x_{i+\tau}\}$.

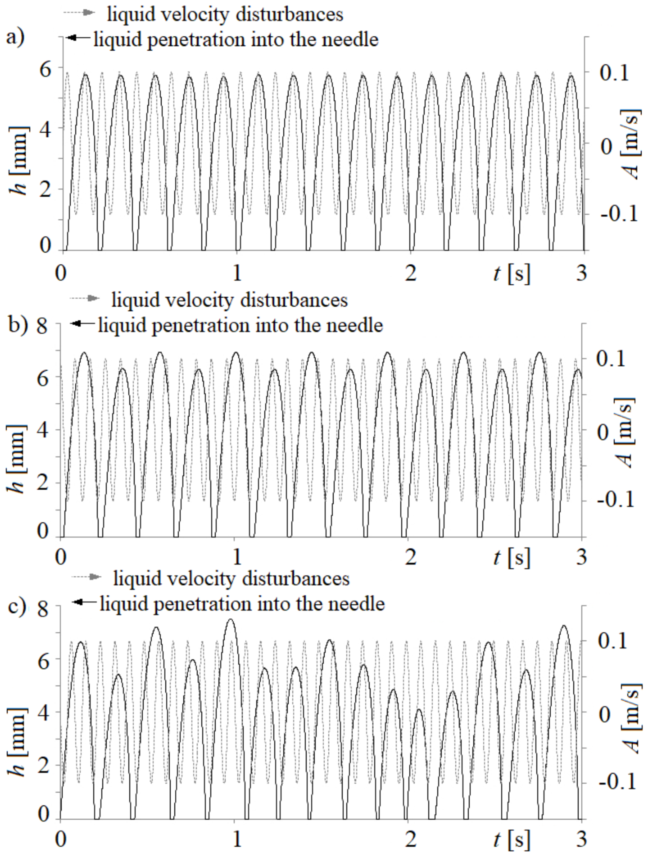


**Fig. 3.** The time series of liquid penetration into the needle obtained from numerical investigations (continuous lines) and liquid velocity perturbations (dotted lines) for selected frequencies of liquid velocity disturbances ($f_{sin}$). a) $f_{sin}$ = 10.02 Hz, b) $f_{sin}$ = 11.52 Hz, c) $f_{sin}$ = 11.9 Hz

In Fig. 4 are shown the reconstructions of 3D attractors for time series of liquid penetration into the needle.

The changes of frequency of occurrence of perturbations in liquid flow above the needle modify the depths of liquid flooding into the needle in successive cycles of bubble departures (Fig.3) and the periodic or chaotic nature of bubble departures (Fig.4). In Fig.3 a the depths of liquid flooding are similar in successive cycles of bubble departures. In this case the trajectories in the 3D attractor reconstruction are close to each other for liquid flooding in subsequent stages of bubble departures (Fig.4 a). In Fig.3 b it can be observed that the changes of the depth of liquid flooding into the needle for successive cycles of bubble departures occurs with two periods. In the first cycle the depth is close to 6 mm and for the second cycle it is close to 7 mm and this process is repeatable in all analysed time series. Two distinct paths overlap in the 3D reconstruction of the attractor (Fig. 4b). As shown in Fig. 3c, the penetration of liquid into the right needle varied unpredictably. The trajectories that compose the 3D attractors are non-repetitive (Fig. 4c).

To verify the timing of bubble departures and occurrence of perturbations of liquid flow above the needle, the frequencies of these phenomena occurrences were estimated. The frequencies were estimated using the FFT method [35,36]. The frequencies of the liquid penetration into the needle and the frequencies of liquid velocity perturbations are shown in Table 2.

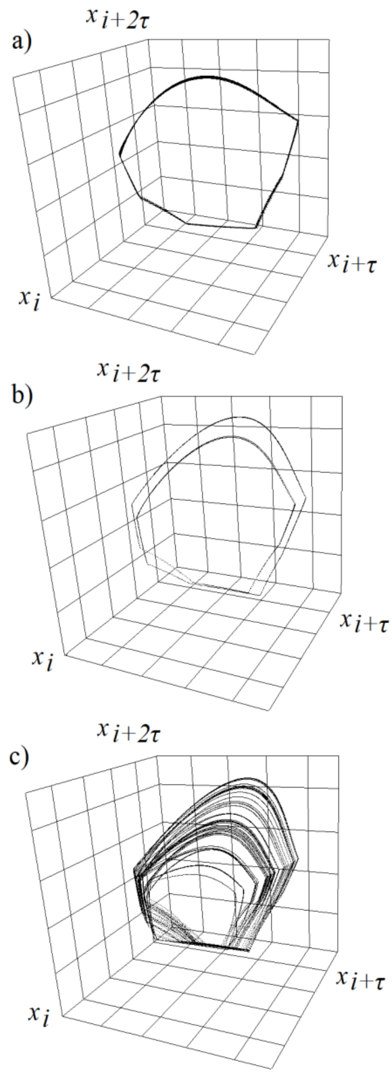


**Fig. 4.** 3D attractors obtained for time series of liquid flow into the needle for selected frequencies of liquid velocity perturbations ($f_{sin}$). a) $f_{sin}$ = 10.02 Hz, b) $f_{sin}$ = 11.52 Hz, c) $f_{sin}$ = 11.9 Hz

**Tab. 2.** The frequencies of the liquid penetration into the needle and the frequencies of liquid velocity perturbations

| | | | |
|---|---|---|---|
| $f_{lp}$ [Hz] | 5.01 | 4.58 | 4.64 |
| $f_{sin}$ [Hz] | 10.02 | 11.52 | 11.90 |
| $f_{lp}/f_{sin}$ [-] | 2.00 | 2.51 | 2.56 |

In the FFT method, the dominant frequency of liquid movement into the needle can be treated as a frequency of bubble departures, but only for periodic changes of depth of liquid penetration into the needle. In the case of chaotic changes of depth of liquid penetration into the needle the dominant frequency is treated as the frequency of the majority of departing bubbles in the analysed time series. Even though this frequency is not fully a frequency of bubble departure, analysis of the dominant frequency can be used for investigations of liquid flow above the needle on bubble departure nature.

To explore how changes in the frequency of liquid velocity above the needle relate to the time series of liquid penetration into the needle, wavelet decomposition analysis and the FFT method were employed. This analysis was performed in Matlab with the Wavelet Toolbox. The Daubechies (db2) method in the Orthogonal Wavelet Family, and I performed five levels of frequency decomposition because the analysed time series have the non-linear character. The signal of details obtained from the 5th level of decomposition was analysed using the Fast Fourier Transform. The power spectra are shown in Fig. 5.

The disturbance frequency of the liquid flow above the needle was marked using fs in Fig.5. In Fig.5 it was shown that liquid flow disturbances modify the nature of the frequency distribution in the analysed time series. If bubble detachment is periodic, then only multiples of the frequency components are observed (Fig.5 a and Fig.5 b). The introduction of non-frequency-synchronized disturbances results in the appearance of multiple frequency components, which determines the chaotic nature of the bubble departure process.

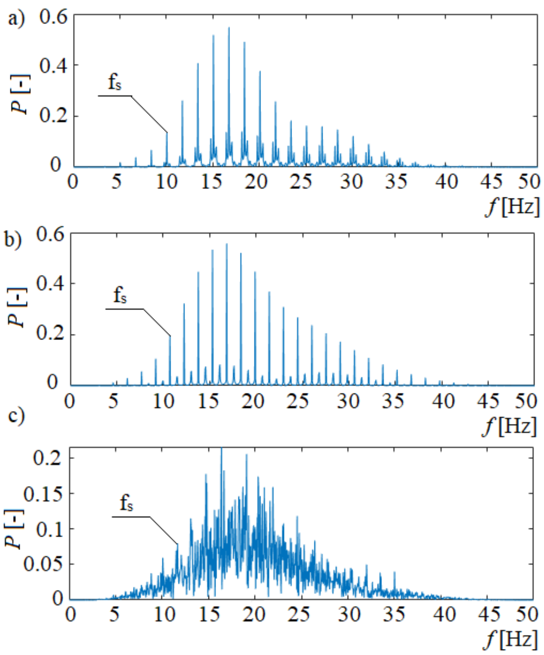


**Fig. 5.** Frequency components obtained from 5th level wavelet decomposition for time series of liquid flow into the needle for selected frequencies of liquid velocity perturbations ($f_{sin}$). a) $f_{sin}$ = 10.02 Hz, b) $f_{sin}$ = 11.52 Hz, c) $f_{sin}$ = 11.9 Hz.

For presented considerations the frequencies of disturbances of liquid velocity above the needle ($f_{sin}$) were chosen so that the ratio of those frequencies and the frequencies of bubbles departures ($f_{lp}$) were: was different from the integer, and the close to half-integer (equal to 2.5) integer (equal to 2). It was shown that synchronisation in the perturbations in liquid flow above the needle and the bubble departures can modify the nature of bubble departures. In the case, when the ratio of $f_{sin}/f_{lp}$ is integer then the depth of liquid flow into the needle changes slightly for successive cycles of bubble departures (Fig.3 a). The trajectories in 3D attractors are close to each other (Fig.4 a). In the case that the ratio of $f_{sin}/f_{lp}$ is integer and half (2.5) then the needle is flooded by the liquid with two consecutive levels of depths of liquid penetration in successive cycles of bubble departures (Fig 3 b). When the ratio of $f_{sin}/f_{lp}$ is other than an integer, then the depth of liquid flooding is varying in the successive cycles of bubble departures.

It can be concluded that the disturbances in liquid flow above the needle modify the conditions in the system supplying gas. Disturbances occurring in liquid flow lead to changes in the depth of liquid penetration and the frequency of bubble detachment. This confirms that adjusting the hydrodynamic conditions above the needle can regulate the process of bubble departure. The results obtained from the numerical model are consistent with the results of the experiment presented in paper [2]. The proposed numerical model containing a component allowing for modification of the velocity and frequency of disturbances in the liquid flow above the needle can be used to numerically study the interaction between the bubbles and the needle the system supplying gas during the bubble departures from two needles.

## 4. CONCLUSIONS

This paper numerically investigates the influence of liquid flow above the needle (modified by departed bubbles) on the bubble departure process. For modelling the interaction the models of the bubble growth and liquid penetration into the needle were used. The numerical model containing the component allows for modification of the velocity and frequency of disturbances in the liquid flow above the needle. This component was developed based on the experimental results presented in this paper [1]. It was shown that the numerical model can be used to study the interaction between the bubbles and the needle gas supply systems during the bubble departures from two needles, because the interaction between the bubbles is related to disturbances in the liquid flow above the needle.

Moreover it was shown that the bubble departure process can be control by adjusting the hydrodynamic conditions above the needle. This is because alterations in the liquid flow and its velocity above the needle affect the boundary conditions in the system supplying gas. The changes in the conditions in the gas supply system correspond with the changes of the periodic or chaotic nature of bubbles departures (visible in time series of liquid penetration into the needle). Consequently, the depth of liquid into the needle and the frequency of bubble departures vary, depending on the occurring disturbances.

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