

HUMAN VISION AS A MULTI-CIRCUIT MATHEMATICAL MODEL OF THE AUTOMATED CONTROL SYSTEM

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Abstract: The paper contains a proposal an original, extended mathematical model of an automatic system of human vision reaction to a forcing light pulse. A comprehensive mathematical model of the vision process was proposed in the form of an equation described in the frequency (dynamics) domain. Mathematical modelling of human senses is very important. It enables better integration of automation systems with a human cooperating with them, also as an automation system. This provides the basis for reasoning based on a mathematical model instead of intuitive reasoning about human reactions to visual stimuli. A block diagram of the proposed system with five human reaction paths is given. The following can be distinguished in the scheme: the main track consisting of: the transport delay of the eye reaction, the transport delay of the afferent nerves, the inertia of the brain with a preemptive action, the transport delay of the centrifugal nerves and the inertial and transport delay of the neuromotor system. In addition, the scheme of the system includes four tracks of negative feedback of motor and force reactions: upper eyelid, lower eyelid, pupil and lens. In the proposed model, the components of each path along with their partial mathematical models are given and discussed. For each reaction path, their overall mathematical models are also given. Taking into account the comprehensive models of all five reaction paths, a complete mathematical model of the automatic system of human reaction to a forcing light impulse is proposed. The proposed mathematical model opens up many possibilities for synchronizing it with mathematical models of many mechatronics and automation systems and their research. Optimizing the parameters of this model and its synchronization with specific models of automation systems is difficult and requires many numerical experiments. This approach enables the design of automation systems that are better synchronized with human reactions to existing stimuli and the selection of optimal parameters of their operation already in the design phase. The proposed model allows, for example, accurate determination of difficulty levels in computer games. Another example of the use of the proposed model is the study of human reactions to various situations generated virtually, for example in flight simulators and other similar ones.

Key words: automation system; human vision; mathematical model

1. INTRODUCTION

Human has five basic senses, each of which has its own specific characteristics and ways of affecting human, in synchronization with other senses. A lot of different papers have been created on the interaction of human and his senses with machines, robots and automation systems. In this regard, for example, the paper of Zhijun et al. [1] on physical human-robot interaction approaches for the developed robotic exoskeleton using admittance control. The model of human interaction with automation was proposed by Parasuraman et al. [2], but it is a proposal of a very simple model. Daley [3] proposes an algorithm development technique that utilizes modes of the human visual system. The basis of the technique evaluation was the Georgia Tech Vision model. Thomas [4] eight models examined as input-output representations of steady-state vision in humans. Proposed three new models for sensitivity. A nonlinear least squares fitting algorithm produced the optimal parameters for each model. Matsui [5] formulates a nonstationary spatio-temporal human vision model has been formulated based on essential properties of the human vision system. Research reported by Karmakar [6] comprises evaluation of pilot's vision in a jet aircraft in virtual environment to demonstrate how vision

analysis tools of digital human modelling software can be used effectively for such study. From present study, it can be concluded that vision analysis tool of digital human modelling software was found very effective. The paper [7] contains a proposal extend model of the human visual system to predict the effects of age. The complete model, including an empirical neural component, can well explain the differences in sensitivity between old and young observers. In the paper [8] a review of Human Visual System (HVS) based Digital Watermarking schemes are presented with their mathematical model.

So far, there have been no scientific inquiries we have come across which treat the human being as a complex automation system.

However, exceptions can be given, such as in a fairly old book by Antoniewicz [9], in which this issue was briefly considered and a model of treating human and his senses as an automatic system was proposed. The paper of Asadi, et al. [10], is also noteworthy which proposed a mathematical model of the vestibular system to ensure successful human perception modelling and simulation.

Mathematical modelling of human senses is of great importance as it enables better integration of the automation system model with the human model cooperating with them as an automation system. It is then possible to combine the above-

mentioned models and build one coherent multi-circuit mathematical model of an automatic system with signals affecting the human senses.

Most biological processes exhibit marked non-linearity in all its kinds and combinations. Both: sensitivity threshold, saturation range, relay action and hysteresis can be observed. These factors describe the behaviour in steady, static, normalized states.

Analyzing the dynamic time of human behaviour observed in the sphere of reaction to external stimuli, one can notice interactions defined in automation as overshoots or higher-order inertia reactions.

In this approach, a group of human behaviours made up of activities related to excessive reaction to external influences should be taken into account. There is usually some redundancy, a quick, violent reaction to stimuli, which expires over time, e.g. complete closure (covering) of the eye, which is then automatically uncovered. In automation, this is a classic differentiation operation. When analyzing human reactions to external stimuli, behaviour control can be assumed using a natural controller analogous to the PID controller. Such processes take place in many aspects of human behaviour. This applies not only to the sense of sight, but also to maintaining balance, controlling pain, effort, resistance to higher temperatures, etc.

Figure 1 presents a diagram of the structural model of human reaction to the received signals developed by the brain after obtaining information from various human senses, proposed by Antoniewicz [9]. The diagram does not show the sensory blocks that acquire information. The causal signal (input to the scheme) is an impulse developed by the cerebral cortex. In the main path there are effectors, i.e. executive organs of the body that perform or change their activity under the influence of a nerve impulse. The main track, as an effector, performs the reaction, after it has been triggered.

The effectors are usually loaded with an external influence. The effectors, in the model view of a human, are his muscles.

The diagram (Fig. 1) illustrates the entire system from the sensor (eye) to the effector (muscles) in functional terms. The system works on the basis of frequency modulation or the number of pulses. The diagram shows the process of responding to a stimulus in terms of impact. It provides both a track for motor reaction and a track for force reaction. It does not indicate in detail

the groups of muscular-defensive tracks related to the structure of the eye system and their protective functions. However, it shows the strengthening track related to the innervations of the muscles, shown in the diagram as a parallel connection. In this way, the innervations of the muscles are depicted. The reinforcing action is shown, generating opportunities related to overcoming the influence of the load, i.e. disturbances for the reaction. Apart from the main track, the scheme has two branches of active control of the regulatory action related to the general concept of force and motor reaction. These tracks represent two functional reaction tracks that are realized in the excitation-human system.

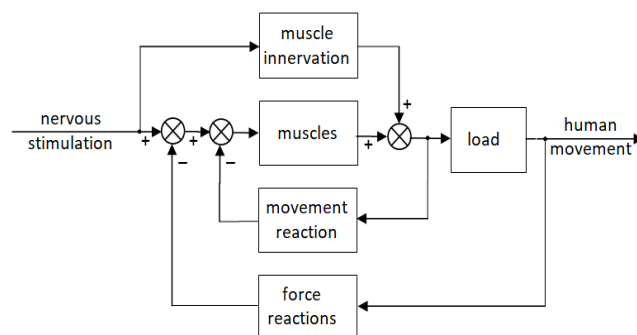


Fig. 1. Block diagram of the excitation pulse system - human reaction [9]

Fig. 2 shows a schematic detailing of the main track without defensive reactions from Fig. 1 of the model structural diagram of the main track of the light signal - human reaction system developed on the basis of Antoniewicz's proposal [9]. The presented system is a series connection, so there are no regulatory actions related to the optimization of the response to the excitation signal. However, it has all the essential terms in the main track with the action command as the output signal. There are no effectors in the diagram, which appear in Fig. 3 in the feedback branches as a reaction to executive and protective actions. To put it simply, it can be assumed that the executive members (effectors) are the last member: the neuromotor system.

In his considerations, Antoniewicz did not propose a mathematical model of the system in Fig. 2.

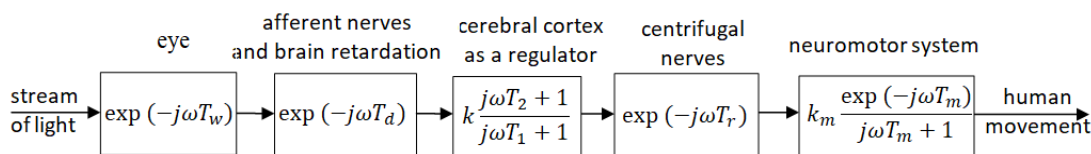


Fig. 2. General equivalent diagram of a human as a controlling operator [9]

2. HUMAN VISION AS MATHEMATICAL MODEL OF THE AUTOMATION SYSTEM

The human visual system is an extremely complex sense. The paper contains a proposal developed spectral mathematical structural model of the system describing the process of the influence of the light signal on the organ of vision and the human neuromotor reaction.

Fig. 3 shows a proposed block diagram of human reaction to a stream of light.

A comprehensive mathematical model determined on the basis of the diagram, shown in Fig.3, is a complex model, having several feedbacks associated with specific groups of effectors. The main track is a supplemented and improved reaction track proposed by Antoniewicz [9]. This circuit consists of the transport delay of the eye reaction, the transport delay of the afferent nerves, the inertia of the brain with anticipatory action, the transport delay of the centrifugal nerves, and the inertial and transport delay of the neuromotor system. It also takes into account the power amplification of signals functionally present in this track.

The diagram of the proposed model of the reaction of human movement to a light excitation signal (Fig. 3) consists of four tracks of negative feedbacks:

- negative feedback track of the motor-force reaction of the upper eyelid,
- negative feedback track of the motor-force reaction of the lower eyelid,
- negative feedback track of the pupil's motor-force reaction,
- negative feedback track of the motor-force reaction of the lens.

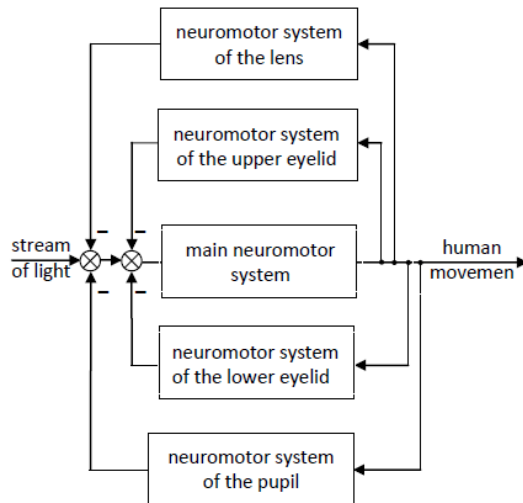


Fig. 3. Block diagram of the light excitation system for human reaction

The diagram shows mathematical models in the frequency domain of the listed components of the main circuit and other circuits.

The block diagram of the light excitation - human reaction system (Fig. 3) is the basis for modelling a complex system. This will enable simulation tests of the model in the time domain as well as in the frequency domain.

Negative feedback track of the motor-force reaction of the upper eyelid is shown in Fig. 4.

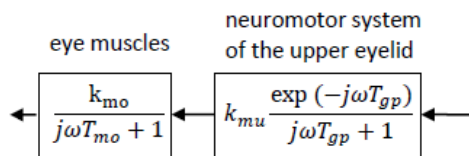


Fig. 4. Negative feedback track block diagram of the motor-force reaction of the upper eyelid

It consists of the neuromotor system of the upper eyelid and the eye muscles. The mathematical model of the negative feedback track of the reaction of the neuromotor system of the upper eyelid and eye muscles has the following form:

$$G_1(s) = k_{mu} \frac{e^{-T_{gp}s}}{T_{gp}s + 1} k_{mo} \frac{1}{T_{mo}s + 1} \quad (1)$$

where: k_{mu} - amplification factor of the neuromuscular reaction system; T_{gp} - time constant associated with the transport delay and inertial action of the upper eyelid; k_{mo} - eye muscle strengthening coefficient; T_{mo} - time constant of inertia of the eye muscles.

Negative feedback track of the motor-force reaction of the lower eyelid is shown in Fig. 5.

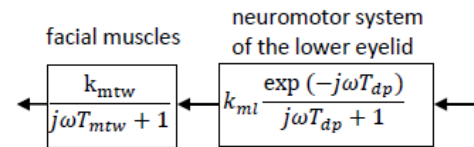


Fig. 5. Negative feedback track block diagram of the motor-force reaction of the lower eyelid

It consists of the neuromotor system of the lower eyelid and facial muscles. The mathematical model of the negative feedback track of the reaction of the neuromotor system of the lower eyelid and facial muscles has the following form:

$$G_2(s) = k_{ml} \frac{e^{-T_{dp}s}}{T_{dp}s + 1} k_{mtw} \frac{1}{T_{mtw}s + 1}, \quad (2)$$

where: k_{ml} - amplification factor of the neuromuscular reaction system; T_{dp} - time constant associated with the transport and inertial delay of the neuromotor system of the lower eyelid; k_{mtw} - coefficient of neuromuscular strengthening of the facial muscles; T_{mtw} - time constant of inertia of facial muscles action.

Negative feedback track of the pupil's motor-force reaction is shown in Fig. 6.

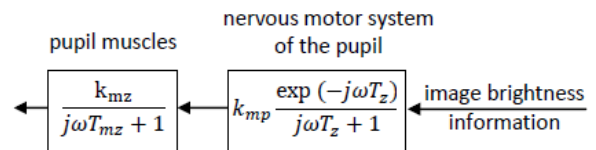


Fig. 6. Negative feedback track block diagram of the pupil's motor-force reaction

It consists of the neuromotor system of the pupil and the muscles of the pupil. The excitation signal is information about the brightness of the image. The excitation signal is information about the sharpness of the image.

The mathematical model of the negative feedback loop of the motor-force reaction of the pupil and the pupillary muscles has the following form:

$$G_3(s) = k_{mp} \frac{e^{-T_zs}}{T_zs + 1} k_{mz} \frac{1}{T_{mz}s + 1}, \quad (3)$$

where: k_{mp} - amplification factor of the neuromuscular reaction system; T_z - time constant related to the transport and inertial delay of the eye pupil; k_{mz} - coefficient of strengthening the pupillary muscles; T_{mz} - time constant of inertia of the action of the muscles of the pupil.

The negative feedback track of the motor-force reaction of the lens consists of the neuromotor system of the lens and the ciliary muscles is shown in Fig. 7.

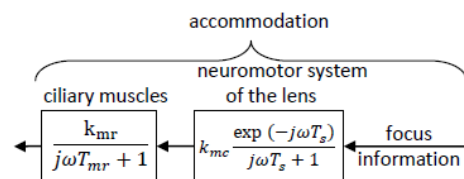


Fig. 7. Negative feedback track block diagram of the motor-force reaction of the lens

The mathematical model of the negative feedback track of the motor-force reaction of the lens and ciliary muscles has the following form:

$$G_4(s) = k_{mc} \frac{e^{-T_s s}}{T_s s + 1} k_{mr} \frac{1}{T_{mr} s + 1} \quad (4)$$

where: k_{mc} - amplification factor of the neuromuscular reaction system; T_s - time constant related to the transport and inertial delay of the lens operation when correcting the sharpness of the image; k_{mr} - ciliary muscle strengthening coefficient; T_{mr} - time constant of inertia of the action of the ciliary muscles.

In Figure 8, showing the detailed block diagram of the light excitation system for human reaction, the basic track can be distinguished. This is a modification of the track proposed by Antoniewicz in Fig. 2. The equation of the proposed main track in algebraic form is as follows:

$$G_0(s) = \frac{k_w e^{-T_w s} k_d e^{-T_d s} k_c (T_2 s + 1) e^{-T_r s} k_r}{T_1 s + 1}, \quad (5)$$

where: k_w - eye amplification factor (overall); T_w - transport delay time (related directly to the eye); k_d - afferent nerve (optical nerve) gain factor; T_d - transport delay time related to the transmission of a signal stimulated by the afferent nerve and brain reaction time; k_c - gain factor of the central part of the nervous system (cerebral cortex); T_2 - lead time related to the self-adjusting regulatory action of the central part of the nervous system; T_r - transport delay time (signal transmission) of the centrifugal nerve; k_r - centrifugal (motor) nerve gain factor; T_1 - time constant of inertia associated with the central part of the nervous system.

The generalized holistic model of human reaction to the light flux, in accordance with the diagram in Fig. 8, has the following form:

$$G(s) = \frac{G_0(s)}{(1 + G_0(s)(G_1(s) + G_2(s) + G_3(s) + G_4(s)))} \frac{k_m e^{-T_M s}}{(T_m s + 1)} \quad (6)$$

where: k_m - amplification factor of the neuromuscular reaction system; T_M - transport delay time associated with the reaction of the neuromuscular system; T_m - time constant of inertia of the neuromuscular system.

The relationship shows a complex system of reactions of the eyesight system consisting of four self-regulating tracks in the protective behaviour of the human eye.

Taking into account the detailed mathematical models of all tracks (1, 2, 3, 4, and 5), the generalized overall model of human reaction to the light stream based on equation (6) has the form of the operator transfer function $G(s)$ generated with the Mathematica program:

$$G(s) = \frac{(e^{-s T_d - s T_M - s T_r - s T_w} k_c k_d k_m k_r k_w (1 + s T_2))}{((1 + s T_1) (1 + s T_m) (1 + (e^{-s T_d - s T_{dp} - s T_{gp} - s T_r - s T_s - s T_w - s T_z} k_c k_d k_{mc} k_{ml} k_{mo} k_{mp} k_{mr} k_{mtw} k_{mu} k_{mz} k_r k_w (1 + s T_2)) / ((1 + s T_1) (1 + s T_{dp}) (1 + s T_{gp}) (1 + s T_{mo}) (1 + s T_{mr}) (1 + s T_{mtw}) (1 + s T_{mz}) (1 + s T_s) (1 + s T_z))))} \quad (7)$$

The work of an eye as a whole is very complicated. The entire system from the eye to the muscles works by modulating the number of pulses or their frequency. This makes it difficult to determine the value of the k_w factor. Its substitute value is close to unity. The muscles work as impulse demodulators.

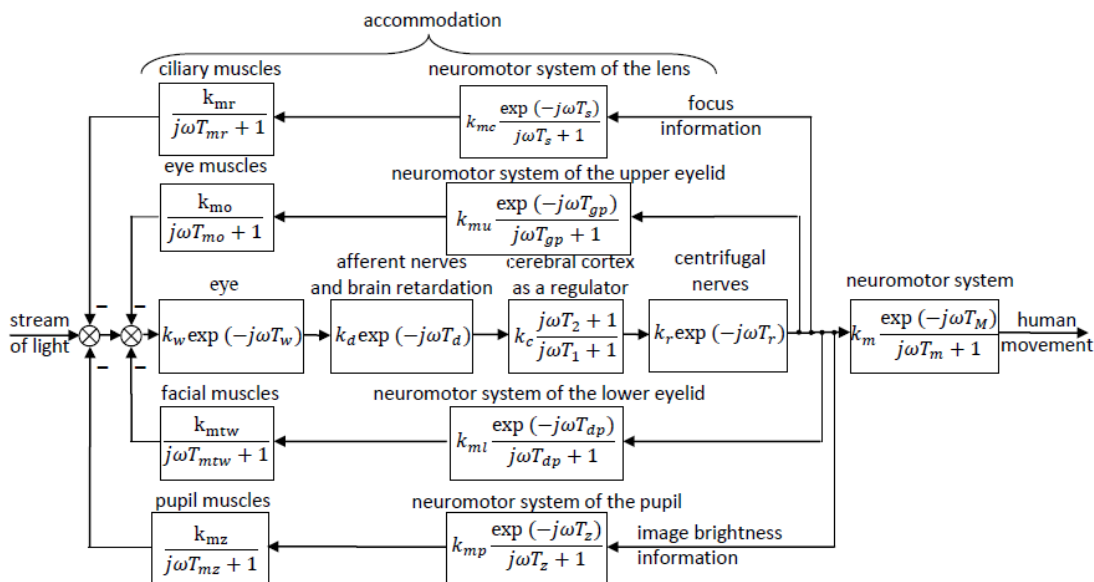


Fig. 8. Detailed block diagram of the light excitation system for human reaction

3. FINAL REMARKS AND CONCLUSIONS

The paper contains a proposal an original, innovative, extended mathematical model of the automatic system of human reaction to a light impulse forcing it. To achieve this goal, a comprehensive mathematical model of the vision process was proposed

in the form of an equation described in the frequency (dynamics) domain. This provides the basis for reasoning based on a mathematical model instead of intuitive reasoning about human reactions to visual stimuli.

Mathematical modelling of human senses is very important. It enables better integration of automation systems with a human cooperating with them, also as an automation system. The ana-

lyzed and described human reactions in a situation of light excitation are self-acting, automatic. Such actions are often referred to as an unconditional reflex.

A block diagram of the proposed system with five human reaction tracks is given. The main track consists of the transport delay of an eye response, the transport delay of the afferent nerves, the inertial action of the brain with anticipation, the transport delay of the centrifugal nerves, and the inertial and transport delay of the neuromotor system. In addition, the system takes into account four tracks of negative feedback of motor and force reactions: upper eyelid, lower eyelid, pupil and lens. In the proposed model, the components of each track along with their partial mathematical models are given and discussed. For each reaction track, their overall mathematical models are also given. Taking into account the models of all five reaction tracks, a complete mathematical model of the automatic system of human reaction to a forcing light impulse is proposed.

Most biological processes exhibit marked non-linearity in all its kinds and combinations. Some of the members, primarily forming the central nervous system and neuromuscular systems, are complex systems with negative feedback and compensation tracks. The system contains many nonlinearities: sensitivity thresholds, ambiguities of positions, overshoots, and the model can be expanded and refined. In practical considerations, it is usually sufficient to assume a more simplified surrogate model for a human.

The proposed mathematical model opens up many possibilities for synchronizing it with mathematical models of many mechatronics and automation systems and their research. Optimizing the parameters of this model and its synchronization with specific models of automation systems is difficult and requires many numerical experiments.

The proposed model is a relatively complicated model. Solving the overall equation of this model and analyzing it is extremely difficult and time-consuming. Simplified partial analyzes of individual branches of the model are possible. This will be continued in the authors' further works. In further research, it is planned to use, among others, genetic algorithms.

The proposed model provides a basis for others to mathematically model issues related to this model. This model and its future solution show a number of aspects of its use. It allows for combining the cooperation of a human and a technical automated system. It will be possible to synchronize the model with the human biological vision system.

Another purpose of the created model is its diagnostic and comparative role.

If the dynamic solution of the human reaction to a light pulse is consistent with the reaction of a representative group of examined people, the presented model can be used as a comparative standard when assessing subsequent people. Another application of the presented model may be related to automatic external support of the quality of vision and support for automatic reflex processes related to the protection of the organs of vision at critical moments of vision.

The mathematical model of the description of the vision process makes it possible to combine it with automation systems in the form of their mathematical models of automation into a unified mathematical model of a complex system. This approach enables the design of automation systems that are better synchronized with human reactions to existing stimuli and the selection of optimal parameters of their operation already in the design phase. The proposed model allows, for example, accurate determination

of difficulty levels in computer games. Another example of the use of the proposed model is the study of human reactions to various situations generated virtually, for example in flight simulators and other similar ones. This approach also makes it possible to adapt models of automation systems to specific, non-standard vision cases. The modeled action therefore includes protective and security processes.

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